Exercises N3, 04.03.2025 - Symmetry, Miller indices, Tensors

1. Using left- and right-handed screws similarly to examples in the lecture, construct a macroscopically homogeneous non-crystalline material of the symmetry $\infty/\ m$.

2. Consider a material of the CsCl-structure (Fig.1) and with the lattice constants a = b = c= 0.4 nm (we call this state of the material A phase). In this material, a phase transition occurs, which leads to a displacement of the central atom of the unit cell along the 4-fold axis parallel to the X_3 axis (as explained in Exercises N2b-1) and in the expansion of the unit cell in the same direction so that a = b = 0.4 nm and c= 0.42 nm (we call this state of the material B phase).

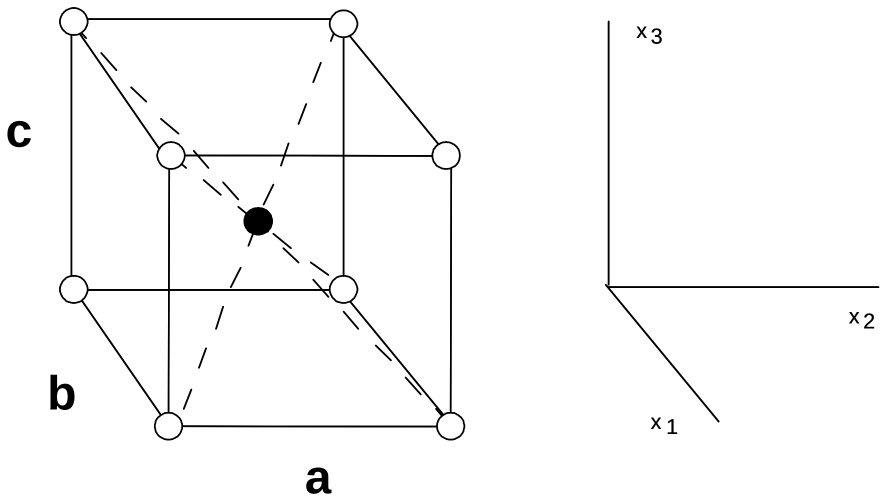

Fig.1

a) In both phases, find the number of crystallographic directions denoted as <011> .
b) In both phases, find the number of different crystallographic planes denoted as {100}.

3. Tensor $T_{ij} = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$

is given in the reference frame (X_1, X_2, X_3) shown in Fig. 2.

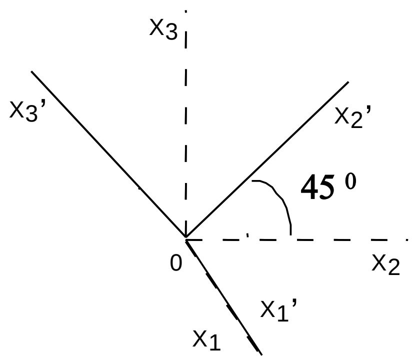

Fig.2

a) Find the table of direction cosines for the frame transformation shown in Fig.2: the rotation by $45^{\underline{0}}$, (X_1, X_2, X_3)→ (X'_1, X'_2, X'_3). Transform this tensor to the (X'_1, X'_2, X'_3) frame using the full tensor calculations.

b) Find the components T'_{11}, T'_{22} and T'_{23} of the tensor transformed to the reference frame T'_{ij} (X_1', X_2', X_3') using the component-product method. Compare results of the methods in **a)** and **b)**.

4. Hook's law relates a second rank stress tensor σ_{ij} and a second rank strain (deformation) tensor ε_{ij} :

$\sigma_{ij} = c_{ijlk}\varepsilon_{lk}$.

Show that c_{ijlk} is a forth rank tensor.